

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9618

Roll No.

--	--	--	--	--	--	--	--	--	--

B. Tech.

End Semester (Third) Examination, 2011-12

MATHEMATICS-III

Time : 3 Hours]

[Total Marks : 100

Note : Attempt *all* questions. Provide the statistical tables which are required by students.

Section-A

Attempt *all* parts of this question. $2 \times 10 = 20$

1. (a) Define analytic function and state the necessary and sufficient condition for function to be analytic.
- (b) Define isolated and non-isolated singular points.
- (c) The fourth moment about the mean of a frequency distribution is 24. What must be the value of standard deviation in order that

the distribution be platykurtic?

- (d) Write the normal equations if the curve $y = a + b\sqrt{x}$ is fitted to $(x_i, y_i), i = 1, 2, \dots, n$.
- (e) Find the parameters p and q of the Binomial distribution whose mean is 9 and variance is $9/4$.
- (f) Write one-way analysis of variance table.
- (g) Construct the forward difference table for $f(x) = x^3 - 2x^2 + 4x + 5$ for $x = 1, 3, 5, 7$.
- (h) What is meant by convergence of iterative methods?
- (i) Define partial pivoting with example.
- (j) What do you mean by initial value problem?

Section-B

2. Attempt any *three* parts of this question : $10 \times 3 = 30$

- (a) Verify Cauchy's theorem for the function $f(z) = 4z^2 + iz - 3$ along the positively oriented square with vertices $(1, 0), (-1, 0), (0, 1)$ and $(0, -1)$.

- (b) Calculate the first four moments about the mean of the following distribution :

x	y
2.0	5
2.5	38
3.0	65
3.5	92
4.0	70
4.5	40
5.0	10

Also find the measures of skewness and kurtosis.

- (c) The following table gives the number of account clerks committing errors and not committing errors among trained and untrained clerks working in an organization.

	No. of Clerks Committing Errors	No. of Clerks not Committing Errors	Total
Trained	70	530	600
Untrained	155	745	900
Total	225	1,275	1,500

Use χ^2 -test to examine the effectiveness of training in preventing errors.

- (d) The equation $2e^{-x} = \frac{1}{x+2} + \frac{1}{x+1}$ has two roots greater than -1 . Calculate these roots correct to five decimal places.
- (e) Use Gauss-Seidel method to solve the following system of equations correct to three decimal places :

$$x + 5y + 54z = 110$$

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

Section-C

Attempt any *two* parts from each question of this Section.

$$5 \times 2 \times 5 = 50$$

3. (a) Determine the analytic function $f(z)$ in terms of z whose real part is $e^{-x}(x \sin y - y \cos y)$.
- (b) State and prove Cauchy's integral formula.
- (c) Use Residue theorem to evaluate :

$$\oint_C \frac{24z - 7}{(z-1)^2(2z+3)} dz,$$

where C is the circle of radius 2 with centre at the origin.

4. (a) Find the moment generating function of the random variable X having the probability density function and hence find the mean for :

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2-x, & 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- (b) Fit a straight line of the form $y = ax + b$ to the following data :

x	y
1	1.0
2	1.2
3	1.8
4	2.5
5	3.6
6	4.7
7	6.6
8	9.1

- (c) Calculate the coefficient of correlation between x and y from the following data :

x	y
1	8
3	12
5	15
7	17
8	18
10	20

5. (a) A die is thrown five times. If getting an odd number is a success, find the probability of getting at least four successes.
- (b) The mean weekly sale of the chocolate bar in candy stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was advertising campaign successful?
- (c) The data of defective of ten samples of size 100 each are given below :

Sample No.	No. of Defectives
1	6
2	9
3	12
4	5
5	12
6	8
7	8
8	16
9	13
10	7

Construct $\bar{n}\bar{p}$ chart and give your comment.

6. (a) Use Regula-Falsi method to find the smallest positive root of the following equation correct to four significant digits :

$$x^3 - 5x + 1 = 0$$

- (b) Express the value of θ in terms of x using the following data :

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

Also find θ at $x = 43$.

- (c) Find an interpolating polynomial to the following data and hence compute $f(1)$:

x	-1	0	3	6	7
$f(x)$	3	-6	39	822	1611

7. (a) The population of a certain town is given below. Find the rate of growth of the population in 1961 :

Year	Population (in Lacs)
1931	40.62
1941	60.80
1951	71.95
1961	103.56
1971	132.65

- (b) Find the approximate value of the following integral using Simpson's rule by dividing the interval into six equal parts :

$$\int_0^x \sqrt{1+3\cos^2 x} dx .$$

- (c) Use Runge-Kutta method of fourth order to find the numerical solution at $x = 0.6$ for :

$$\frac{dy}{dx} = \sqrt{x+y}, \quad y(0.4) = 0.41$$